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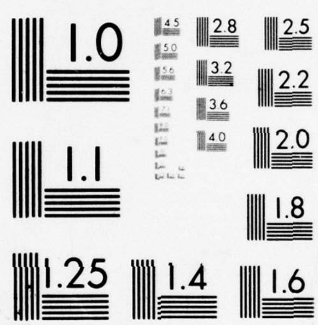

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Analysis of Altitude Errors Arising from Area Target Returns in FM Altimeters

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Search Radar Branch
Radar Division

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A theoretical analysis is presented for the errors, in a conventional and servoed slope FM altimeter, caused by the use of finite beamwidth antennas over an area target. In our analysis, an area target is considered to be a level surface as far as general physical characteristics are concerned, but is "rough" as far as acting as a backscatterer of energy. Errors in altitude measurements are determined analytically for different terrain models. Error curves are shown as a function of antenna beamwidth. The results show that a positive error exists for both types of altimeters and the error

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20. Abstract (Continued)

increases with the increase in antenna beamwidth. For a given antenna beamwidth, the servoed slope altimeter has smaller errors compared to those of a conventional FM altimeter, especially when the terrain reflectivity is independent of the incidence angle. The difference between the two altimeters becomes small as the antenna beamwidth decreases. For the case of sea surface as a target, the difference between the two altimeters is negligible and the increase in errors with the increase in antenna beamwidth is small. For this reason, it may be advantageous to use a wider antenna beamwidth (small antenna) especially when antenna stability is a problem, provided the antenna beamwidth is not so wide as to violate the assumption of level surface due to sea surface wave structure. For a situation where the altimeter antenna is directed at an angle α from the vertical, the error in general increases with α, more so for a conventional altimeter. However, the increase is quite small for broadbeam antennas. This result also points to the advantage of using broadbeam antennas.

alpha

CONTENTS

ANALYSIS OF ALTITUDE ERRORS ARISING FROM AREA TARGET RETURNS IN FM ALTIMETERS

INTRODUCTION

A Radio Altimeter is a special radar system which measures altitude by employing either pulse or frequency modulation (FM). In pulse systems, the time interval between the transmitted pulse and its echo is a direct measure of range. Hence, for low altitude applications a very narrow pulse radar system is required. Therefore, pulse type altimeters are usually preferred for high altitude measurements and FM altimeters are usually preferred for low altitude measurements. For this reason, FM altimeters are being considered for the measurement of the sea surface profile from a surface effect ship. Consequently, the study here deals with FM altimeters. In many FM altimeters a triangular modulation with wide-deviation FM at a fixed low modulation frequency is usually employed. In such systems, the altitude is directly proportional to the difference frequency between transmitted and echo signals. Another FM system, which has been used widely in recent years, employs a feedback loop to keep the difference frequency constant, irrespective of the altitude, and varies the period of the modulation repetition rate in direct proportion to altitude. It is the purpose of this report to study the errors introduced by area targets in these two FM systems, as a function of antenna beamwidth and the reflecting properties of the terrain surface, and compare their performance. This error, sometimes referred to as "terrain averaging error," is present only when the antenna beamwidth is finite. Williamson [1] did an extensive analytical study on the errors introduced by area targets in a conventional FM altimeter. For completeness, part of his work is included in this report. There are other error sources, such as quantization error, errors due to transmitter leakage, and multipath error, which exist in some FM/CW altimeters, and which are discussed by King, et.al [2] and others [3,4]. However, quantization errors do not exist and other errors have negligible effects in the servoed slope altimeter [4]. For this reason those errors are not discussed in this report. They must, however, be taken into account or their effects be reduced in any practical application of conventional altimeters for low-altitude height finding.

BASIC FORMULATION

In the FM Radio Altimeters considered in this report, the transmitted power is kept constant and the frequency is varied linearly with time, as in a triangular sawtooth or a sawtooth waveform. The basic formulation is the same for both types of modulations. For that reason, only the triangular modulation will be considered in this section. Figure 1 shows the frequency versus time of the transmitted and received signals for a time delay τ, which is proportional to the altitude h of a target. From Fig. 1, the following relation can be obtained directly.

$$f = \tau \frac{\partial F}{\partial t} \tag{1}$$

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Substituting $\frac{\partial F}{\partial t} = \frac{\Delta F}{T}$ and $\tau = \frac{2h}{c}$ in (1) and rearranging, one obtains

$$h = \frac{c}{2} \frac{f_b}{\Delta F} T \tag{2}$$

where

τ = the transit time

T = half period of the triangular modulation

f_b = beat frequency (or difference frequency)

h = altitude to be measured

and c is the velocity of light.

Equation (2) is the basic formula used in all modern FM radio altimeters. However, it is used differently in different altimeters.

CONVENTIONAL FM RADIO ALTIMETER [5]

In a conventional FM altimeter, equation (2) is employed with T and ΔF kept constant. The altitude h is therefore proportional to the difference frequency f_b.

The block diagram of a conventional radio altimeter is shown in Fig. 2. For the present, the received signal is assumed to be the return from a single point reflector. After mixing the received signal with a portion of the transmitter signal, the low frequency component of the mixer output is selected, amplified in a receiver with appropriate bandwidth and applied to a counter which, for instance, counts the number of beat frequency cycles during the full period $2T$. This counter displays the altitude which is proportional to the difference frequency f_b.

For a triangular frequency modulation, an exact analysis of the system described above will result in a difference frequency spectrum containing many harmonic lines, even for a point target. However, if the modulation frequency is low, the lines close to f_b are very predominant in the spectrum. Furthermore, the zero-crossing rate of the difference signal is still nearly equal to $2f_b$ with negligible error. Hence, representation of the difference signal spectrum as a single line at f_b is justified because of its greater simplicity.

Errors Arising from Area Targets:

Since the interest here is in the performance of the FM altimeters in the presence of area targets, it is assumed that the received signal is due to reflections from a large number of scatterers of independent cross sections and random ranges from the altimeters. The latter property results in the phases of the individual returns being random, and the corresponding difference frequencies also being random in nature. If no single reflector is large compared to the rest, the situation strongly implies that the received signal will be gaussian since it consists of the superposition of a large number of signals which are independent in both frequency and phase.

When the difference signal is no longer essentially a single sinusoid, a theoretical expression for the average zero-crossing rate is no longer easy to determine. However, when the difference signal is gaussian, Williamson[1] showed that the average number of (positive and negative going) zero crossings per second is given by

$$\bar{n} = \frac{1}{\pi}\left[\frac{\int_0^\infty \omega^2 F(\omega)\,d\omega}{\int_0^\infty F(\omega)\,d\omega}\right]^{1/2} \tag{3}$$

where

$F(\omega)$ is the power spectral density of the difference signal.

The limiting case of a point target is also covered by (3) since then $F(\omega) = (1/2\ \pi)\ \delta(\omega-\omega_b)$ and has value only at $\omega=\omega_b$. The result of applying this to (3) is seen to be $\bar{n} = \omega_b/\pi = 2f_b$, which is the correct answer. An equation for $F(\omega)$, which depends on the antenna beamwidth, gain, pointing direction and the reflecting properties of the target, will be obtained in a later section.

Now, suppose that it is desired to measure the altitude (shortest range) from an area target to the altimeter location. If the true difference frequency corresponding to this range is f_o, the measured difference signal must have $2f_o$ zero crossings per second on the average or there will be an error. The average error $\bar{\varepsilon}$ is the quantity

$$\bar{\varepsilon} = \bar{n} - 2f_o \tag{4}$$

the percent of error is given by

$$\% \ \bar{\varepsilon} = 100 \left[\frac{\bar{n}}{2f_o} - 1 \right] \tag{5}$$

SERVOED SLOPE FM RADIO ALTIMETER [3,4]

In this altimeter, the frequency deviation ΔF is kept the same and the difference frequency f_b is maintained constant irrespective of altitude using a feedback loop. The only variable parameter proportional to altitude is the duration T of the modulation cycle. Therefore, equation (2) can be written as

$$h = AT \tag{6}$$

where

$$A = \frac{c}{2} \frac{f_b}{\Delta F}$$

Fig. 3 shows the asymmetrical modulation used in the present altimeter. Fig. 4 shows the block diagram of a servoed slope altimeter. Instead of a counter as is used in conventional FM system, there is a tracking frequency discriminator tuned to the desired beat frequency f_b. The spectrum of the beat frequency remains centered on this frequency since the error voltage provided by the tracking discriminator drives the modulation rate of the transmitter (or duration T of the modulation cycle since the frequency deviation ΔF is constant).

A control discriminator ensures that the beat frequency spectrum is centered on f_b. In the absence of a spectrum, a search cycle is triggered and the alarms are displayed after appropriate time constants. Searching implies sweeping through the various modulation rates, from the lowest altitude up to the highest limit. As soon as the modulation rate corresponds to the actual altitude, the energy content in the control discriminator rises above a preset threshold and the altimeter loop is locked and will continuously track the altitude variation.

A qualitative discussion on the altitude errors due to area targets is given in [3,4]. It is the purpose of this section to obtain analytical expressions for those errors.

Errors Arising from Area Targets:

First consider a return signal from a single point with the corresponding difference frequency f. If f differs from the center frequency f_c of the discriminator, there will be an error voltage applied to the modulator which changes the modulation period. This feedback action continues until the received difference frequency $f = f_c$ or $f-f_c = 0$. The corresponding modulation period determines the correct altitude. However, for an area target, the return signal contains a sum of the returns of a large number of independent scatterers (within the antenna beamwidth) and the total return power is the sum of the components. This is essentially due to the fact that the phase angle associated with the return from each scatterer is uniformly distributed and is statistically independent of those of the other returns. Making use of the above concept, the operation of the discriminator and the feedback loop is given by the following equation [6]

$$\int_0^{\infty} A(r)\ [f-f_c]\ dr = 0 \tag{7}$$

where

A(r) dr is the incremental power received from the area between the ranges r and r+dr

f is the received difference frequency from a target at slant range r when the measured modulation period is T_m

f_c is the center frequency of the discriminator. It is also the correct difference frequency for a target at range h and correct modulation period T_c.

It is shown in a later section that the incremental power A(r) dr can also be expressed by $F(\omega)\ d\omega$, where $F(\omega)$ is the power spectal density of the received signal when the modulation period is T_m and $\omega = 2\pi f$. Substituting $F(\omega)\ d\omega$ for A(r) dr in (7), we have

$$\int_0^{\infty} F(\omega)\ [f-f_c]\ d\omega = 0 \tag{8}$$

From the definitons of f, f_c, T_m, T_c and equation (2), we have

$$f = \frac{2r}{c}\ \frac{\Delta F}{T_m} \tag{9}$$

and

$$f_c = \frac{2h}{c}\ \frac{\Delta F}{T_c} \tag{10}$$

Similarly, the received difference frequency f_o for a range h when the modulation period is T_m is given by

$$f_o = \frac{2h}{c} \frac{\Delta F}{T_m} \tag{11}$$

The following relation is obtained using (10) and (11)

$$f_c = f_o \frac{T_m}{T_c} \tag{12}$$

Substituting for f_c in (8) using (12), we have

$$\int_o^\infty F(\omega) \left[f - f_o \frac{T_m}{T_c}\right] d\omega = 0 \tag{13}$$

Therefore, the equation for the measured modulation period T_m can be obtained from (13) as

$$T_m = T_c \frac{\int_o^\infty F(\omega) \left(\frac{\omega}{\omega_o}\right) d\omega}{\int_o^\infty F(\omega)\, d\omega} \tag{14}$$

Since the measured and correct altitudes are proportional to T_m and T_c respectively, the percentage altitude error due to an area target is gien by

$$\% \,\bar{\varepsilon} = \left(\frac{T_m - T_c}{T_c}\right) \times 100 \tag{15}$$

Substituting for T_m in (15) using (14), we have

$$\% \,\bar{\varepsilon} = \left[\frac{\int_o^\infty F(\omega) \left(\frac{\omega}{\omega_o}\right) d\omega}{\int_o^\infty F(\omega)\, d\omega} - 1\right] \times 100 \tag{16}$$

POST MIXER SPECTRAL DENSITY DUE TO AN AREA TARGET:

Since it is impossible to construct antennas with which all energy could be directed vertically downward, the error caused by returns from other than ground zero (i.e. directly beneath the altimeter) must be determined. In our analysis, an area target is considered to be a level surface as far as general physical characteristics are concerned, but is "rough" as far as acting as a backscatterer of energy. Fig. 5 shows a typical small patch, ΔA, illuminated by an altimeter located at an altitude h from the level surface. The video power received by the altimeter from that small patch is

$$\Delta P_r = \frac{A_o A_c G^2 \sigma_o \Delta A}{r^4} \tag{17}$$

where $A_o = P_t \lambda^2/(4\pi)^3$ is a system constant

A_c = is a system gain constant

P_t = transmitted power

G = antenna gain in the direction of the patch (both transmitter and receiver antennas are assumed to have identical gain characteristics)

λ = wavelength

σ_o = radar cross section per unit area of the patch

r = slant range to the patch

The size of the patch is assumed to be small enough that the range to all points in the patch is approximately equal to r. However, the video signal due to the return from this patch will contain a number of sinusoidal components in the angular frequency range ω to $\omega+\Delta\omega$, corresponding to the ranges r and $r+\Delta r$.

The summation of power from all patches with the same range r will yield the total video power in the incremental interval ω to $\omega+\Delta\omega$ with ω being functionally related to r as

$$r = \frac{cT\omega}{4\pi\Delta F} \tag{18}$$

Eq. (18) is obtained using Eq. (2) by replacing h by r and f_b by $\omega/2\pi$.

Thus, the video power in the small interval ω to $\omega+d\omega$ is given by

$$P(r) = \frac{A_o A_c}{r^4} \int_o^{2\pi} G^2(\theta,\phi)\ \sigma_o(\theta)\ \rho d\rho d\phi \tag{19}$$

where

$$\theta = \cos^{-1}\left(\frac{h}{r}\right),\ \phi = \text{azimuth angle and } \rho = \sqrt{r^2-h^2} \tag{19a}$$

Since $\rho d\rho$ = rdr, one can write

$$P(r) = A(r)\ dr \tag{20}$$

where

$$A(r) = \frac{A_o A_c \sigma_o(\theta)}{r^3} \int_o^{2\pi} G^2(\theta,\phi)\ d\phi \tag{21}$$

Since $F(\omega)$ is the power spectral density of the received signal, the total received power in a small interval ω to $\omega+d\omega$ can also be given by

$$P(r) = F(\omega)\ d\omega \tag{22}$$

Equating (20)and (22) gives

$$F(\omega) = A(r)\ \frac{dr}{d\omega} \tag{23}$$

Substituting for A(r) from (21) and $(dr/d\omega)$ from (18) in (23), one obtains

$$F(\omega) = K\left(\frac{\omega_o}{\omega}\right)^3 \sigma_o(\theta) \int_o^{2\pi} G^2\ (\theta,\phi)\ d\phi \tag{24}$$

where $K = \frac{TA_oA_cc}{2\Delta Fh^3}$ and ω_o is the video signal received from range h. In obtaining (24), the following relation which may be obtained from (18), is used

$$\frac{r}{h} = \frac{\omega}{\omega_o} \tag{24a}$$

It may be noted that θ can be expressed in terms of ω using (19a) and (24a) as

$$\theta = \cos^{-1}(\omega_o/\omega) \tag{25}$$

It can be seen from (24) and (25) that $F(\omega)$ can exist only for $\omega \geq \omega_o$, the equality giving the frequency of that part of the video signal due to returns from directly beneath the altimeter.

Normal Incidence:

When the antenna is directed vertically downward, the antenna pattern is independent of ϕ and Eq. (24) becomes

$$F(\omega) = K\left(\frac{\omega_o}{\omega}\right)^3 G^2\left(\cos^{-1}\frac{\omega_o}{\omega}\right)\sigma_o\left(\cos^{-1}\frac{\omega_o}{\omega}\right) \tag{26}$$

Oblique Incidence:

When the antenna is directed at an angle away from the vertical, the integral in Eq. (24) may not have a closed form solution. However, it is possible to solve it numerically. For a special case of sea surface as a target, it will be shown in a later section, that an approximate analytical solution is possible.

NUMERICAL RESULTS AND COMPARISON

In this section, numerical results will be given for the altitude errors in the two types of altimeters which may be expected due to terrain averaging (area targets). The results will be presented for different types of level surfaces, including the sea surface and for different values of antenna beamwidths when the antenna is pointed vertically down (normal incidence). The case of oblique incidence is also considered in the case of the sea surface.

Level Ground: Constant σ_o

For the purpose of numerical computation, the two-way radiation pattern of the altimeter antenna is assumed to be of the form

$$G(\theta) = \cos^n(\theta) \tag{27}$$

or

$$G\left(\cos^{-1}\frac{\omega_o}{\omega}\right) = \left(\frac{\omega_o}{\omega}\right)^n \tag{28}$$

Substituting the value of G^2 from (28) and a constant value for σ_o in (26) and then in (3) and noting that $\omega \geq \omega_o$, one obtains the equation for the average number of zero crossings as

$$\bar{n} = 2f_o \left[\frac{\int_1^{\infty} \left(1/W\right)^{2n+1} dW}{\int_1^{\infty} \left(1/W\right)^{2n+3} dW} \right]^{\frac{1}{2}} \tag{29}$$

where $W = (\omega/\omega_o)$.

In general, the upper limit in (29) is not infinity but depends on the filter bandwidth used in the receiver and the altitude to be measured. If ω_m is the highest frequency allowed to pass through the filter, the upper limit of W is $W_m = (\omega_m/\omega_o)$. For a given ω_m, the upper limit is larger for smaller ω_o (smaller altitude). Hence, when the altimeter is required to measure very small altitudes, the upper limit may be taken as infinity with the understanding that the result obtained is the worst case which applies for low altitudes and the percentage of error for higher altitudes will be smaller.

Evaluating the integrals in (29) and substituting $\bar{n}$ in (5), one obtains the percent of error ε_c in the conventional altimeter as

$$\varepsilon_c = \left[\sqrt{1 + \frac{1}{n}} - 1 \right] \times 100 \quad \% \tag{30}$$

Following a similar procedure for the case of the servoed slope altimeter, one can show that the modulation period T_m given in equation (14) becomes

$$T_m = T_o \frac{\int_1^{1+B} \left(1/W\right)^{2n+2} dW}{\int_1^{1+B} \left(1/W\right)^{2n+3} dW} \tag{31}$$

In this case the upper limit of $W = \frac{\omega}{\omega_o}$ is taken as 1+B, where B is the fractional bandwidth of the filter used in the receiver. In the case of the servoed slope altimeter, ω_o is a constant. Therefore, the upper limit for W depends only on B and not on the altitude to be measured. Therefore, the percent of error is independent of the altitude.

Evaluating the integrals in (31) and substituting in (16), one obtains the percent of error ε_s for the case of a servoed slope altimeter as

$$\varepsilon_s = \left\{ \frac{(2n+2)(1+B)}{2n+1} \left[\frac{1-(1+B)^{2n+1}}{1-(1+B)^{2n+2}} \right] -1 \right\} \times 100\% \tag{32}$$

In the numerical computation B is assumed to be 0.2 which corresponds to the altimeter model AHV-6 [7].

Fig. 6 shows the computed results of the errors given by (30) and (32) as a function of antenna beamwidth. The corresponding values of n are also shown on the horizontal axis. From Fig. 6 it may be noted that the errors decrease as n is increased (antenna beamwidth decreased). Also the errors are smaller for the servoed slope altimeter. Therefore, when the altimeters are required to cover the very low altitudes, the servoed slope FM altimeter has a decided advantage over the conventional FM altimeter when σ_o is a constant. The possibility of using a very narrow antenna beamwidth is not very practical as it requires a very stable platform for the altimeter location.

Sea Surface: Normal Incidence

The main motivation of the work reported here is to study the terrain averaging error in two types of altimeters when they are used to measure the sea surface wave profile from a surface effect ship. First, we shall consider the case when the altimeters are directed vertically downward (normal incidence).

In order to determine the power spectral density, the radar cross section σ_o per unit area should be known for the sea surface. Since the altimeters operate in 4 to 5 GHz range, the experimental data obtained by Guinard and Daley [8,9] is used in obtaining an approximate exponential form

$$\sigma_o = D \cos^2(\theta)\, e^{-A\sin\theta} \tag{33}$$

where A and D are constants and θ is the incidence angle. For our purpose, it is not necessary to know the exact value of D. Fig. 7 shows the measured [8] and approximate σ_o given by (33) with A=10, as a function of θ, the angle of incidence. Both the curves are almost coincident up to $\theta=35^o$ and the small deviation for $\theta>35^o$ will not be of much consequence as σ_o is already down by 25 dB compared to the value at $\theta=0$.

Again, assuming the antenna pattern to be of the form $G(\theta) = \cos^n\theta$, the average number of zero crossings, as given by (3) becomes

$$\bar{n} = 2f_o \left[\frac{\int_{\omega_o}^{\infty} \left(\frac{\omega_o}{\omega}\right)^3 \left(\frac{\omega_o}{\omega}\right)^{2n} e^{-A\sin\theta} \, d\omega}{\int_{\omega_o}^{\infty} \left(\frac{\omega_o}{\omega}\right)^5 \left(\frac{\omega_o}{\omega}\right)^{2n} e^{-A\sin\theta} \, d\omega} \right]^{\frac{1}{2}} \tag{34}$$

Noting that $\cos\theta = \omega_o/\omega$ and letting $\sin\theta = x$, it can be shown that

$$\bar{n} = 2f_o \left[\frac{\int_o^1 x(1-x^2)^n \; e^{-Ax} \, dx}{\int_o^1 x(1-x^2)^{n+1} \; e^{-Ax} \, dx} \right]^{\frac{1}{2}} \tag{35}$$

For a given n, the integrals given in (35) can be expressed as a summation of the known integrals of the form

$$\int_o^1 x^m e^{-Ax} dx = \frac{1}{A^{m+1}} \gamma\,(m+1,A) \tag{36}$$

where γ is the incomplete gamma function [10] and is given by the series

$$\gamma\,(m+1,A) = m! \left[1-e^{-A}\left(\sum_{k=o}^{m} \frac{A^k}{k!} \right) \right] \tag{37}$$

Having determined $\bar{n}$ using (35), the percentage of altitude error in the conventional altimeter is obtained using equation (5) and is plotted in Fig. 8 as a function of the antenna beamwidth. The corresponding value of n is also shown.

Following the above procedure for the servoed slope altimeter, the equation (14) for T_m takes the following form for the sea surface

$$T_m = T_o \frac{\int_o^{.55277} x\sqrt{1-x^2}\ (1-x^2)^n\ e^{-Ax}\ dx}{\int_o^{.55277} x(1-x^2)^{n+1}\ e^{-Ax}\ dx} \tag{38}$$

where the upper limit of the integrals is determined for a fractional bandwidth B=0.2 of the filter in the receiver.

Since $0 < x < .55277$, $\sqrt{1-x^2}$ can be approximated by $1-0.5x^2$. Then, for a given value of n, the integrals in (38) can be expressed as a summation of integrals of the form

$$\int_o^U x^m e^{-Ax}\ dx = \frac{1}{A^{m+1}}\ \gamma\ (m+1,\ AU) \tag{39}$$

where γ is again an incomplete gamma function as defined in (37).

Having determined T_m using (38), the percentage of altitude error in the servoed slope altimeter is obtained using (15) and is plotted in Fig. 8 for comparison with that of the conventional altimeter. From these results, it may be noted that the antenna beamwidth has negligible effect on the altitude errors. This is due to the fact that the sea surface returns are attenuated at a rapid rate with increase in incidence angle. This has the effect of acting as a very narrow band filter. Therefore, as long as the antenna beamwidth is not too small, it has a negligible effect on the altitude errors. Also, it may be noted from Fig. 8 that the errors in the two altimeters are comparable. This is due to the fact that the errors are mainly determined by the reflecting properties of the sea surface rather than by any differences in the two systems.

Sea Surface: Oblique Incidence

When the altimeter antennas are not pointed vertically downward,

the power spectral density of the return signal is given by equation (24). However, the antenna pattern will not be symmetrical with respect to ϕ. Fig. 9 shows the geometry of the oblique incidence case. The antennas are pointed at an angle α from the vertical and in the direction of z'. Let θ' and ϕ' be the polar coordinates corresponding to the rectangular coordinates x', y' and z'. The polar coordinates θ and ϕ correspond to the rectangular coordinates x, y and z. In the prime coordinates, the antenna pattern will have symmetry with respect to ϕ' and can be expressed as $\cos^n(\theta')$.

Using the coordinate transformation, one can obtain a relation between θ', θ and ϕ and the antenna pattern can be expressed in terms of θ and ϕ as

$$G(\theta,\phi) = \cos^n(\theta') = (\cos\theta \cos\alpha - \sin\theta \sin\phi \sin\alpha)^n \tag{40}$$

Substituting the antenna pattern given by (40) in (24) and performing the integration with respect to ϕ, one can obtain the power spectral density for oblique incidence. However, in general, this cannot be done because the antenna patterns are defined only for $\theta' \leq \pi/2$ and the integration limits on ϕ will not be from 0 to 2π for $\theta' \geq \pi/2 - \alpha$. Fortunately, for the case of the sea surface, the reflected ray contribution decreases quite rapidly as the angle of incidence increases. For this reason, the effect on the results will be negligible if the limits of ϕ are taken to be 0 to 2π, when α is not too large. Therefore, the power spectral density for the sea surface and oblique incidence is given by

$$F(\omega) = K\cos^2(\theta)\left(\frac{\omega_o}{\omega}\right)^3 e^{-A\sin\theta} \int_0^{2\pi} (\cos\theta\cos\alpha - \sin\theta\sin\phi\sin\alpha)^n d\phi \tag{41}$$

Knowing $F(\omega)$, equations (5) and (16) are used to obtain the altitude errors in the two types of altimeters using a procedure similar to that used for the normal incidence case except that the equations become a lot more complicated and lengthy, especially for large n. For this reason, the detailed procedure used in obtaining the computed errors given in Fig. 10 is not included here. In Fig. 10 the percentage of altitude errors is plotted as a function of beam pointing angle α for antenna beamwidths of 65.5°, 90° and 120°. For larger beamwidths (small antennas) the error does not change much with α. As the antenna beam narrows, the error increases with increase in α for both the altimeters except that the increase in error is slightly more for the conventional altimeter compared to that for the servoed slope altimeter.

CONCLUSIONS

A theoretical analysis is presented for the errors in a conventional and a servoed slope FM altimeter, caused by the use of a finite beamwidth antenna over an area target. In our analysis, an area target is considered to be a level surface as far as general physical characteristics are concerned, but is "rough" as far as acting as a backscatterer of energy. The analysis and numerical examples considered in this report show that for a given antenna beamwidth, the servoed slope altimeter has smaller errors compared to those of a conventional FM altimeter, expecially when the terrain reflectivity is independent of the incidence angle. The difference between the two altimeters becomes small as the antenna beamwidth decreases. The altitude error increased with the increase in antenna beamwidth. However, for the case of sea surface, the difference between the two altimeters is small and the increase in errors with the increase in beamwidth is quite small. In such a case it may be advantageous to use a wider antenna beamwidth (small antenna), especially when the antenna stability is a problem. However, for proper sea surface profiling the antenna beamwidth cannot be so wide as to violate the assumption of a level surface.

In the case of a sea surface, when the altimeter antenna is directed at an angle α from the vertical, the errors in general increase with the angle α, more so for a conventional altimeter than for a servoed slope altimeter. However, the increase was quite small for broadbeam antennas. This result also points to the advantage of using broadbeam antennas.

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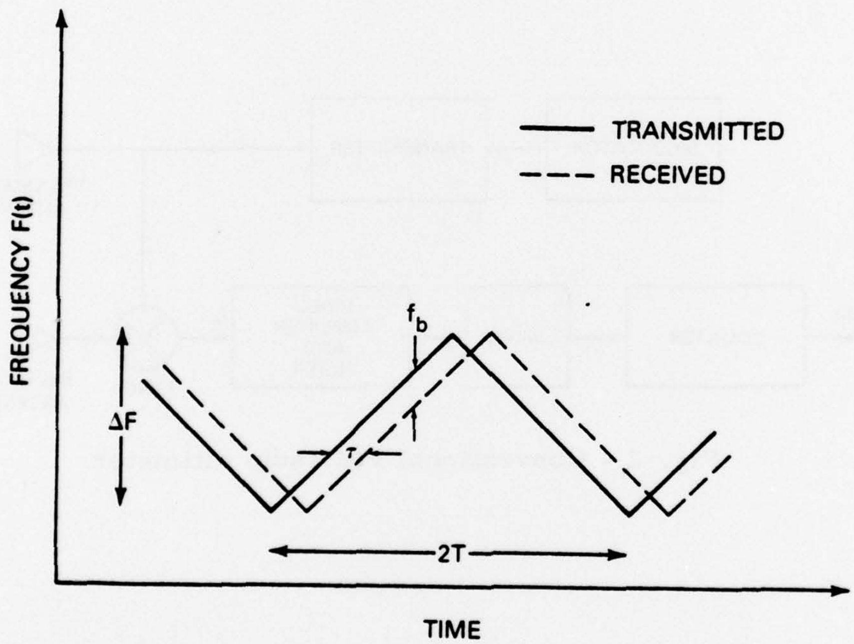

Fig. 1 - Transmitted and received signal frequencies vs time for periodic triangular modulation

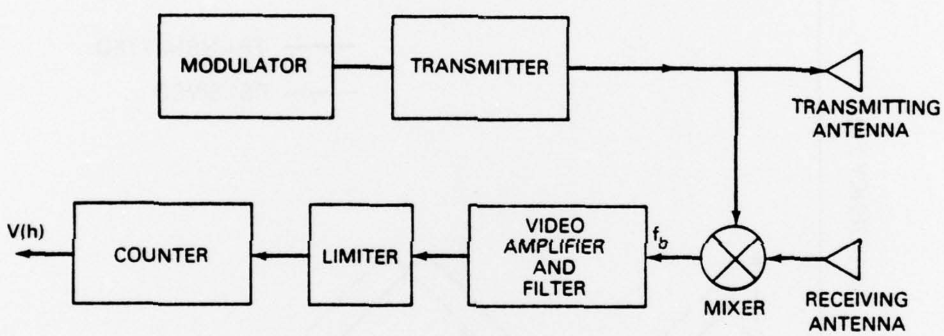

Fig. 2 - Conventional FM radio altimeter

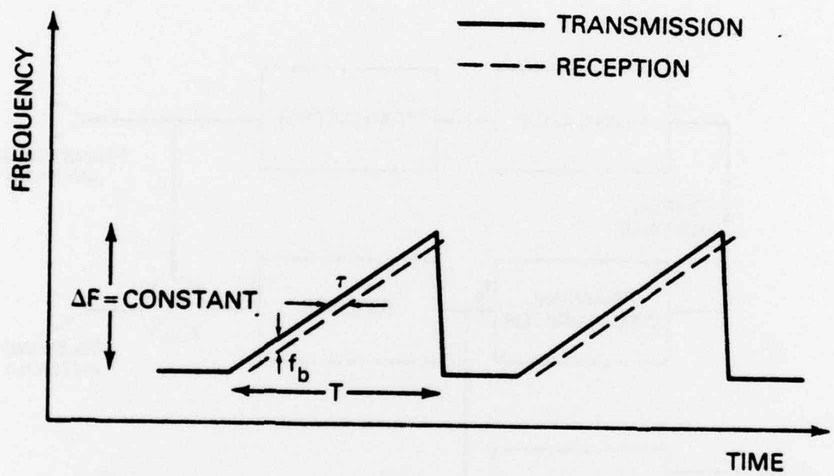

Fig. 3 - Asymmetrical modulation used in servoed slope altimeter

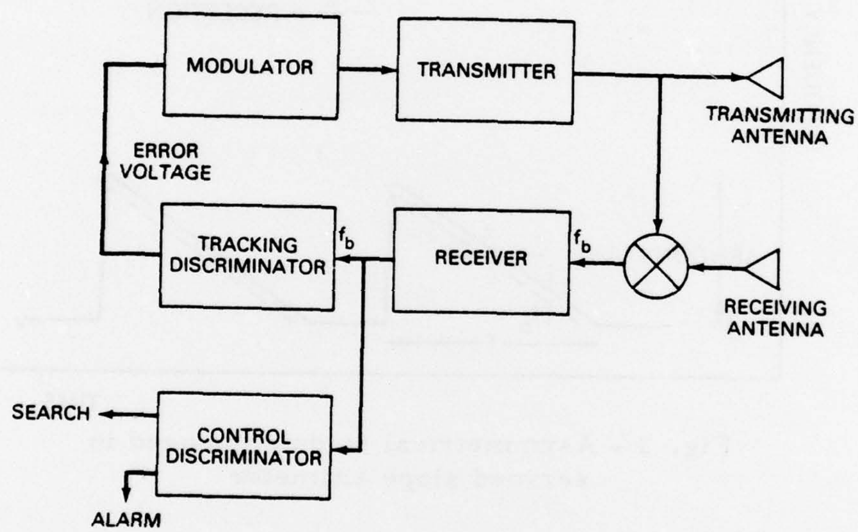

Fig. 4 - Servoed slope FM altimeter

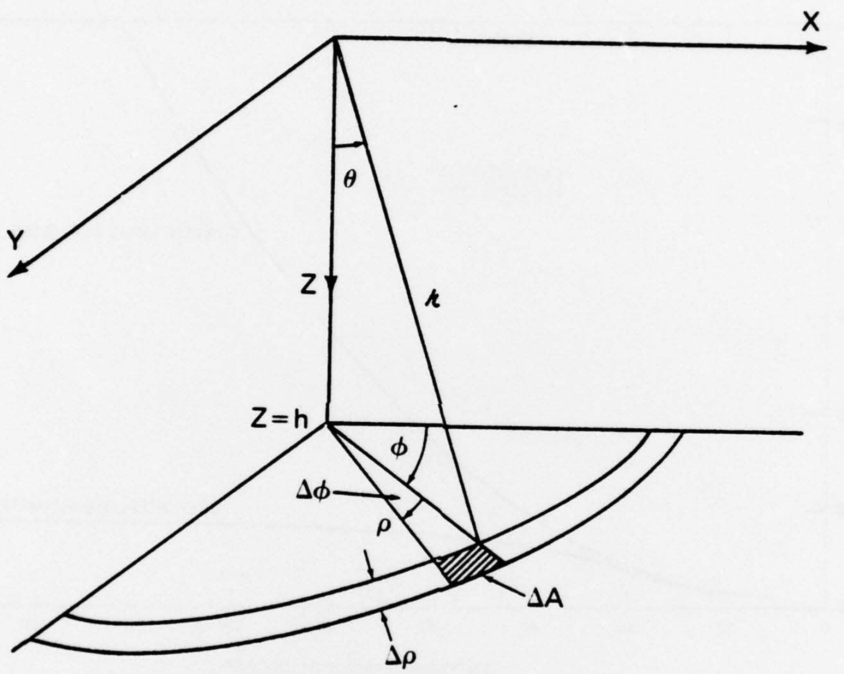

Fig. 5 - Area target geometry

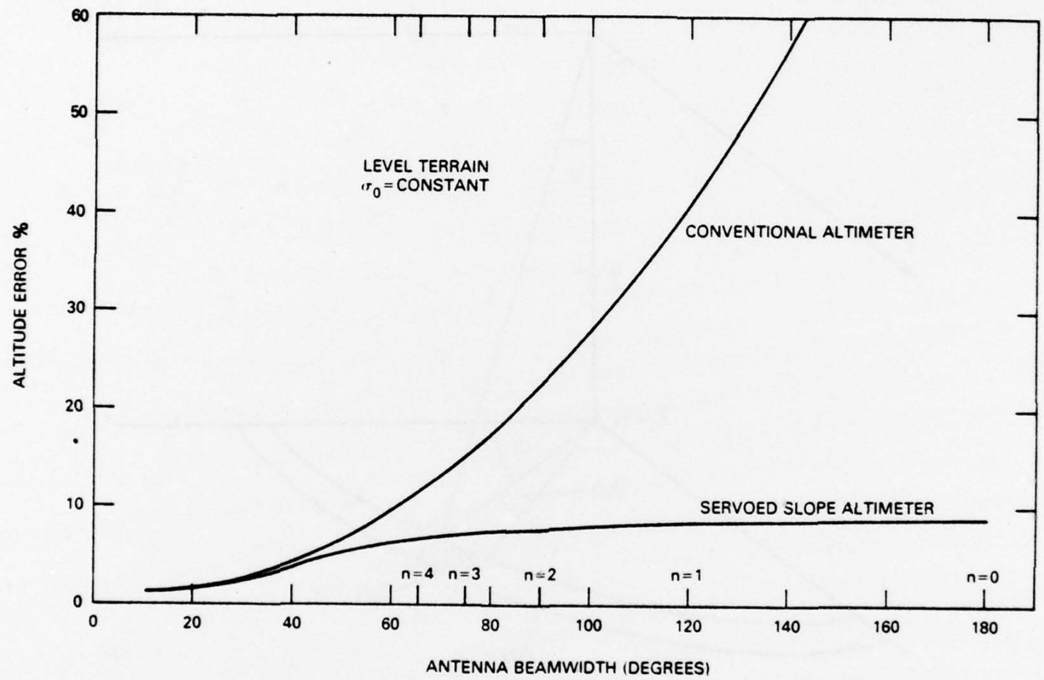

Fig. 6 - Altitude error vs antenna beamwidth for two altimeters

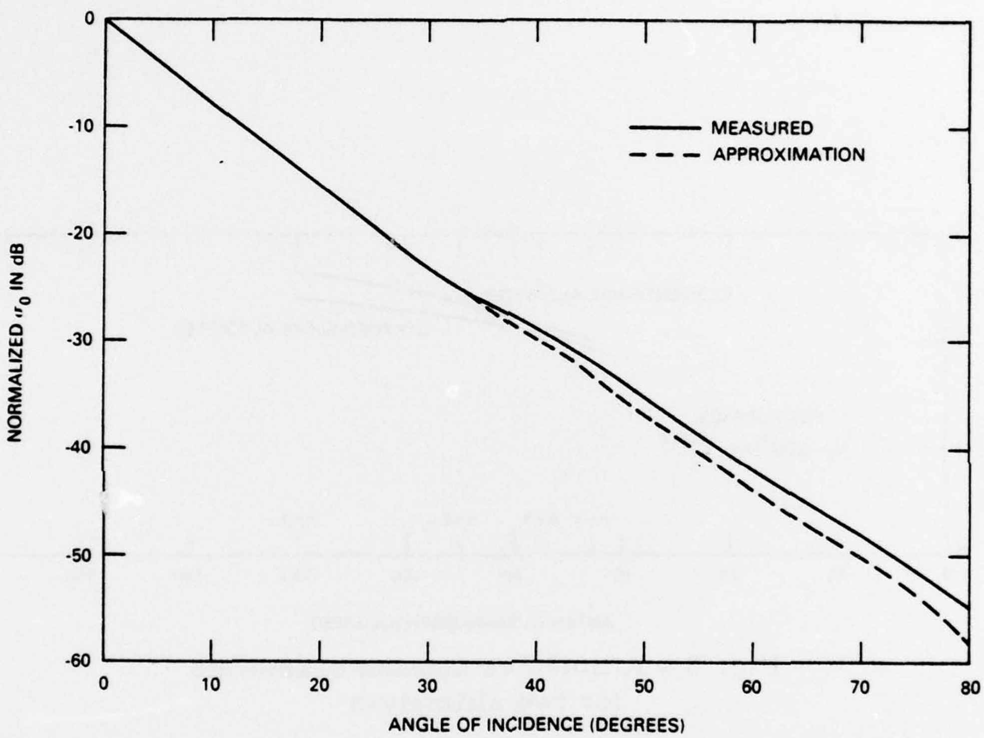

Fig. 7 - Measured sea surface radar cross section σ_0 and an approximation

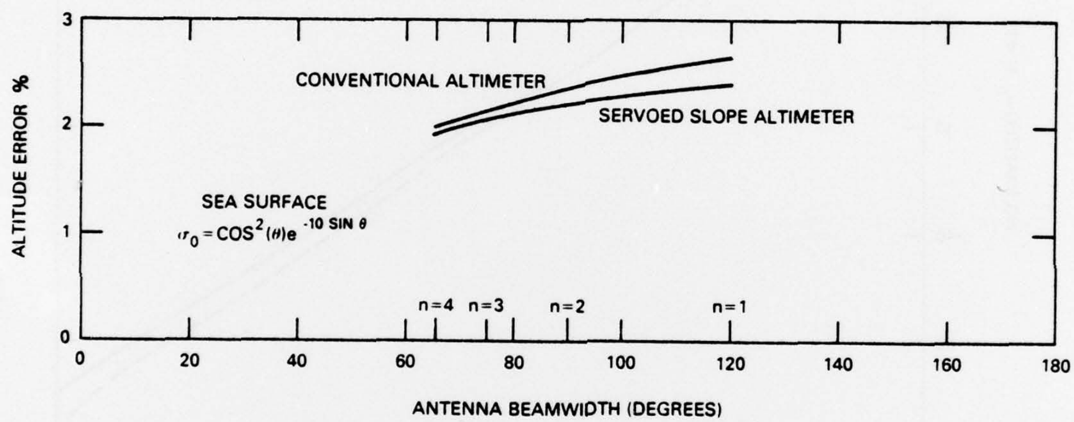

Fig. 8 - Altitude vs antenna beamwidth for two altimeters

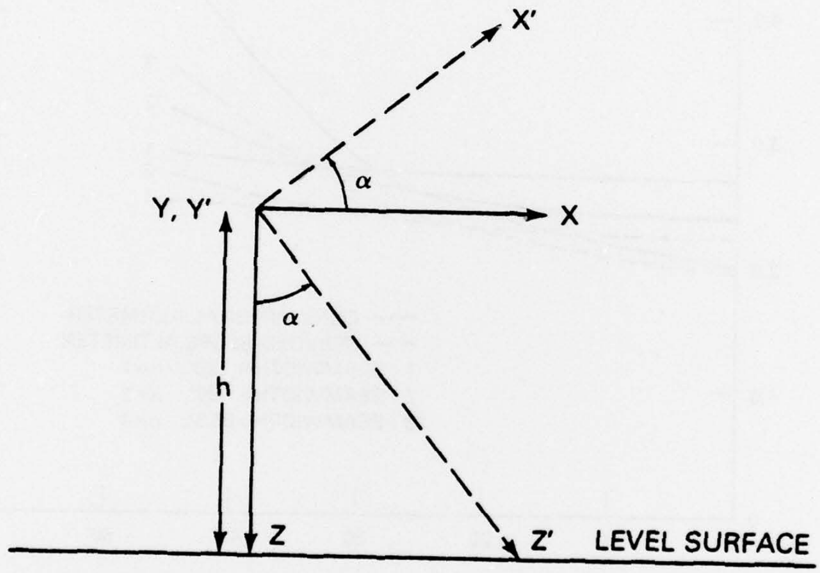

Fig. 9 - Coordinate system used for oblique incidence

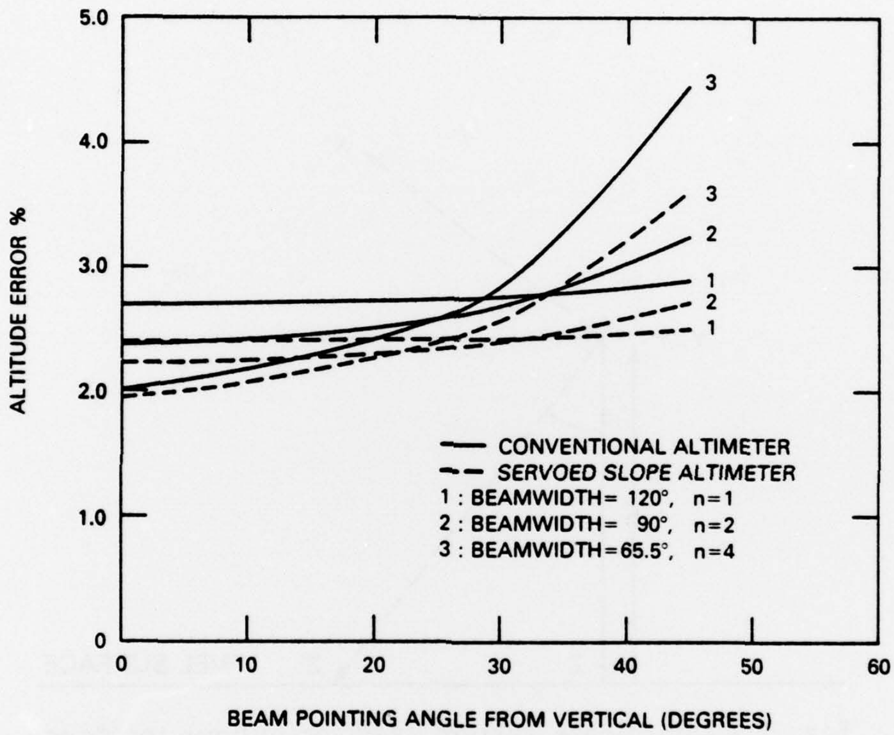

Fig. 10 - Altitude error for oblique incidence, sea surface with $\sigma_0 = \cos^2 \Theta \, e^{-10 \sin\Theta}$